Math 10A with Professor Stankova
Worksheet, Discussion #28; Monday, 10/30/2017
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Probability

Example

1. Suppose that the probability density function P that an atom emits a gamma wave satisfies the following differential equation $P' = -10P$ for $t \geq 0$ and $P(t) = 0$ for $t < 0$. Find P and calculate the CDF associated with P.

2. For the above PDF, find the probability that a gamma wave is emitted from -1 seconds to 1 second.

Problems

3. True False Since the CDF is an antiderivative of the PDF, there are multiple CDFs for a given PDF (and they differ by a $+C$).

4. True False The area underneath a CDF must be equal to 1.

5. True False A PDF must be continuous.

6. True False Let $P(x) = Cx^3$ for $-1 \leq x \leq 2$ and 0 otherwise. Since $\int_{-1}^{3} P(x)dx = C(16 - 1/4)$, setting $C = (16 - 1/4)^{-1}$ makes P into a PDF.

7. Let $P(x) = Cx^2(10 - x)$ on $0 \leq x \leq 10$ and 0 otherwise. Find C such that P is a PDF and its corresponding CDF. Find the probability that we choose a number between 0 and 1.

8. Let $P(x) = C(x - 1)(x + 1)$ on $-1 \leq x \leq 1$ and 0 otherwise. Find C such that P is a PDF and its corresponding CDF. Find the probability that we choose a number between 0 and 1.

9. Let $P(x)$ satisfy $\frac{dP}{dx} = 2x$ for $0 \leq x \leq 1$ and $P(x) = 0$ otherwise. Find P such that it is a PDF and its corresponding CDF. Find the probability that we choose a number between 0 and 1.

10. Let $F(x) = \frac{x-1}{x+1}$ for $x \geq 1$ and 0 for $x \leq 1$. Show that F is a CDF. Find the PDF associated with it and the probability that we choose a number between 1 and 2.

11. Find numbers A, B such that $A \arctan(x) + B$ is a CDF and find the PDF associated with it. Find the probability that we choose a number between 0 and 1.

12. Let $F(x) = \ln x$ for $1 \leq x \leq a$ and $F(x) = 0$ for $x \leq 1$ and $F(x) = 1$ for $x \geq a$. Find a such that F is a continuous CDF and find the PDF associated with it. Find the probability that we choose a number between 1 and 2.

Logistic Growth

Example

13. The rate of growth of a population is logistic with an intrinsic rate of growth of $r = 10$ and a carrying capacity of 1000. Write this down as a differential equation. Solve for the population if the initial population is 100.

Problems

14. The rate of growth of a population is logistic with an intrinsic rate of growth of $r = 10$ and a carrying capacity of 1000. Assume that the population is harvested at a rate proportional to its population with a constant of proportionality of 5. Write this down as a differential equation. What is the fate of the population for different initial sizes?

15. The rate of growth of a population is logistic with an intrinsic rate of growth of $r = 10$ and a carrying capacity of 1000. Assume that the population is harvested at a rate proportional to its population with a constant of proportionality of 10. Write this down as a differential equation. What is the fate of the population for different initial sizes?

16. The rate of growth of a population is logistic with an intrinsic rate of growth of $r = 10$ and a carrying capacity of 1000. Assume that the population is harvested at a rate proportional to its population with a constant of proportionality of 15. Write this down as a differential equation. What is the fate of the population for different initial sizes?

17. The rate of growth of a population is logistic with an intrinsic rate of growth of $r = 10$ and a carrying capacity of 1000. Assume that 2100 individuals are killed every year. Write this down as a differential equation. What is the fate of the population for different initial sizes?

18. The rate of growth of a population is logistic with an intrinsic rate of growth of $r = 10$ and a carrying capacity of 1000. Assume that 2500 individuals are killed every year. Write this down as a differential equation. What is the fate of the population for different initial sizes?

19. The rate of growth of a population is logistic with an intrinsic rate of growth of $r = 10$ and a carrying capacity of 1000. Assume that 2900 individuals are killed every year. Write this down as a differential equation. What is the fate of the population for different initial sizes?